ABSTRACT

To measure enzyme kinetics model of a catalytic base where an enzyme E reacts with a substrate S to form a complex ES. The ES complex can be dissociated: either by one enzyme molecule E with one substrate molecule S or either one enzyme molecule E with one product molecule P; we can use the model established by Michaelis and Menten.

$$E_f + S \underset{k_r}{\overset{k_f}{\rightleftarrows}} ES \xrightarrow{k_{cat}} P + E_f$$

E_f : Enzyme free
S : Substrate
ES : Complexe
P : Product

$k_f > 0$, $k_r \geq 0$ and $k_{cat} > 0$ are the rate constants of the different reactions.

k_f : kinetic forward constant
k_r : kinetic reverse constant
k_{cat} : kinetic catalytic constant

The rate of the enzymatic reaction (*v)* is the rate of the occurence product.

$$v = \frac{dP}{dt} = k_2[ES](t)$$

If the substrate concentration is infinite, all the free enzyme form ES complex.
Now, if [E_f] = [E_{total}] so $v = v_{max}$.

$$v_{max} = k_2[E_{total}]$$

Whether, $\frac{v}{v_{max}} = \frac{k_2[ES](t)}{k_2[E_{total}]}$ and give **the equation of speed :** $\boxed{v = v_{max}\frac{[ES](t)}{[E_{total}]}}$

<u>Define Michaelis Constant or *Km*</u>

To set the *Km*, define steady state of *ES(t)*

$$\frac{d[ES](t)}{dt} = 0$$

$$\frac{d[ES](t)}{dt} = k_f[E_f](t)[S](t) - (k_r + k_{cat})[ES](t)$$

Whether, $k_f[E_f](t)[S](t) - (k_r + k_{cat})[ES](t) = 0$

Michaelis constant : $\boxed{\frac{[E_f](t)[S](t)}{[ES](t)} = \frac{k_r + k_{cat}}{k_f} = K_m}$

<u>Define Equation of MICHAELIS - MENTEN</u>

It is known, $\frac{[E_f](t)[S](t)}{[ES](t)}=K_m$ and $[E_f](t)=[E_{total}]-[ES](t)$. By substitution,

$$\frac{([E_{total}]-[ES](t))[S](t)}{[ES](t)}=\left(\frac{([E_{total}])}{[ES](t)}-1\right)S(t)=\frac{[E_{total}]S(t)}{[ES](t)}-S(t)=K_m$$

Solve,

$$\frac{[E_{total}]S(t)}{[ES](t)}=K_m+S(t)\Rightarrow\frac{[E_{total}]}{[ES](t)}=\frac{K_m+S(t)}{S(t)} \quad \Rightarrow \boxed{\frac{[ES](t)}{[E_{total}]}=\frac{S(t)}{K_m+S(t)}} \quad \textit{(1)}$$

By substitution, (1) is integrated in the equation of speed and we get **the equation of MICHAELIS - MENTEN** :

$$\boxed{v=v_{max}\frac{S(t)}{K_m+S(t)}}$$

The numerical model of Michaelis - Menten

We can deduce the differential relation :

$$[P]'(t)=\frac{v_{max}[S](t)}{K_m+[S](t)}$$

Differential equation of the instantaneous velocity of product formation.

$$[S]'(t)=-\frac{v_{max}[S](t)}{K_m+[S](t)}$$

Differential equations of the instantaneous speed of substrate disappearance.

$$[ES](t)=[E_{total}]\frac{S(t)}{K_m+S(t)}$$

Differential equation of the instantaneous velocity of complex formation.

$$[E_f](t)=[E_{total}]-[ES](t)$$

Differential equation of the instantaneous velocity of complex disappearance or enzyme free.

We get a possible solution for this model with Euler-Cauchy method of taking a step h> 0 but small enough.

As the variable S (t) is defined by the recursion : $\boxed{S_{i+1}=S_i-h\frac{v_{max}S_i}{Km+S_i}}$ (2)

And P_i is define by the recursion : $P_i = P_0 + S_0 - S_i$

Limite of Euler-Cauchy method

For define the approximation to numerical solution of equation by Euler-Cauchy method, it is necessary to get a time step h quite small such as $h > 0{,}1$ (empirical determination)
If h is not quitte small, so the numerical solution for equation become a degenerate solution for $\varepsilon \simeq \lim S(t)$

Relative error of the Eule-Cauchy method applied to the Michaelis-Menten model or Er (t)

For everything t with two different steps, such as $h_\alpha > h_\beta$.
With

$$Er_{h_\alpha}(t) = 1 - \frac{S(t) - h_\beta \dfrac{v_{max} S(t)}{K_m + S(t)}}{S(t) - h_\alpha \dfrac{v_{max} S(t)}{K_m + S(t)}}$$

Either h_α, $S_{i+1} = S_i - h_\alpha \dfrac{v_{max} S_i}{Km + S_i}$ and for h_β, $S_{i+1} = S_{i+\frac{h_\alpha}{h_\beta}} - h_\beta \dfrac{v_{max} S_{i+\frac{h_\alpha}{h_\beta}}}{Km + S_{i+\frac{h_\alpha}{h_\beta}}}$

Integration in our model

The product of degradation by NB-esterase 13 are ethylène glycol and terephthalic acid. Ethylene glycol diffuses into the cell through the plasma membrane and terephthalic acid. Terephthalic acid enter in the cell using the TPA transporter.

S_0: Substrat in media
k_{diff}: Diffusion constant

$$S_0 \xrightarrow{k_{diff}} \boxed{\text{Michaelis-Menten système}}$$

Model for diffusion throug the plasma cell or S_{cell}(t)

At the order of magnitude of cell $S_0 = \infty$.
S_{cell}(t) is define by addition of the diffusion constant k_{diff} at the recursion (2).

So S_{cell}(t) is define by the recursion

$$\boxed{S_{cell\, i+1} = k_{diff} + S_i - h \frac{v_{max} * S_i}{K_m + S_i}}$$